

## **APPENDIX A: MATHEMATICAL SUMMARY STATEMENT FOR THE STANDARD CGE MODEL**

<b>SETS</b>		
$\alpha \in A$		activities
$\alpha \in ACES(\subset A)$		activities with a CES function at the top of the technology nest
$\alpha \in ALEO(\subset A)$		activities with a Leontief function at the top of the technology nest
$c \in C$		commodities
$c \in CD(\subset C)$		commodities with domestic sales of domestic output
$c \in CDN(\subset C)$		commodities not in CD
$c \in CE(\subset C)$		exported commodities
$c \in CEN(\subset C)$		commodities not in CE
$c \in CM(\subset C)$		imported commodities
$c \in CMN(\subset C)$		commodities not in CM
$c \in CT(\subset C)$		transactions service commodities
$c \in CX(\subset C)$		commodities with domestic production
$f \in F$		factors
$i \in INS$		institutions (domestic and rest of the world)
$i \in INSD(\subset INS)$		domestic institutions
$i \in INSDNG$ $(\subset INSD)$		domestic nongovernment institutions
$h \in H(\subset INSDNG)$		households

### **PARAMETERS**

<b>Latin Letters</b>		
$cwts_c$		weight of commodity $c$ in the <i>CPI</i>
$dwts_c$		weight of commodity $c$ in the producer price index
$ica_{c a}$		quantity of $c$ as intermediate input per unit of activity $a$
$icd_{c c'}$		quantity of commodity $c$ as trade input per unit of $c'$ produced and sold domestically
$ice_{c c'}$		quantity of commodity $c$ as trade input per exported unit of $c'$
$icm_{c c'}$		quantity of commodity $c$ as trade input per imported unit of $c'$
$int_a$		quantity of aggregate intermediate input per activity unit

$iva_a$	quantity of value-added per activity unit
$\overline{mps}_i$	base savings rate for domestic institution $i$
$mps01_c$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
$pwe_c$	export price (foreign currency)
$pwm_c$	import price (foreign currency)
$qdst_c$	quantity of stock change
$\overline{qg}_e$	base-year quantity of government demand
$\overline{qinv}_c$	base-year quantity of private investment demand
$shif_{if}$	share for domestic institution $i$ in income of factor $f$
$shii_{i'v}$	share of net income of $i'$ to $i$ ( $i' \in INSDNG$ ; $i \in INSDNG$ )
$t\alpha_a$	tax rate for activity $a$
$te_e$	export tax rate
$tf_f$	direct tax rate for factor $f$
$\overline{tins}_i$	exogenous direct tax rate for domestic institution $i$
$tins01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
$tm_c$	import tariff rate
$tq_c$	rate of sales tax
$trnsfr_{if}$	transfer from factor $f$ to institution $i$
$tva_a$	rate of value-added tax for activity $a$
<b>Greek Letters</b>	
$\alpha_a^a$	efficiency parameter in the CES activity function
$\alpha_a^{va}$	efficiency parameter in the CES value-added function
$\alpha_a^{ac}$	shift parameter for domestic commodity aggregation function
$\alpha_c^q$	Armington function shift parameter
$\alpha_c^t$	CET function shift parameter
$\beta_{ach}^h$	marginal share of consumption spending on home commodity $c$ from activity $a$ for household $h$
$\beta_{ch}^m$	marginal share of consumption spending on marketed commodity $c$ for household $h$
$\delta_a^a$	CES activity function share parameter
$\delta_{ac}^{ac}$	share parameter for domestic commodity aggregation function
$\delta_c^q$	Armington function share parameter
$\delta_c^t$	CET function share parameter
$\delta_{fa}^{va}$	CES value-added function share parameter for factor $f$ in activity $a$
$\gamma_{ch}^m$	subsistence consumption of marketed commodity $c$ for household $h$
$\gamma_{ach}^h$	subsistence consumption of home commodity $c$ from activity $a$ for household $h$

$\theta_{ac}$	yield of output c per unit of activity a
$\rho_a^a$	CES production function exponent
$\rho_a^{va}$	CES value-added function exponent
$\rho_c^{ac}$	domestic commodity aggregation function exponent
$\rho_c^q$	Armington function exponent
$\rho_c^t$	CET function exponent
<b>EXOGENOUS VARIABLES</b>	
$\overline{CPI}$	consumer price index
$\overline{DTINS}$	change in domestic institution tax share (= 0 for base; exogenous variable)
$\overline{FSAV}$	foreign savings (FCU)
$\overline{GADJ}$	government consumption adjustment factor
$\overline{IADJ}$	investment adjustment factor
$\overline{MPSADJ}$	savings rate scaling factor (= 0 for base)
$\overline{QFS}_f$	quantity supplied of factor
$\overline{TINSADJ}$	direct tax scaling factor (= 0 for base; exogenous variable)
$\overline{WFDIST}_{fa}$	wage distortion factor for factor f in activity a
<b>ENDOGENOUS VARIABLES</b>	
$DMPS$	change in domestic institution savings rates (= 0 for base; exogenous variable)
$DPI$	producer price index for domestically marketed output
$EG$	government expenditures
$EH_h$	consumption spending for household
$EXR$	exchange rate ( $LCU$ per unit of $FCU$ )
$GOVSHR$	government consumption share in nominal absorption
$GSAV$	government savings
$INVSHR$	investment share in nominal absorption
$MPS_i$	marginal propensity to save for domestic non-government institution (exogenous variable)
$PA_a$	activity price (unit gross revenue)
$PDD_c$	demand price for commodity produced and sold domestically
$PDS_c$	supply price for commodity produced and sold domestically
$PE_c$	export price (domestic currency)
$PINTA_a$	aggregate intermediate input price for activity $a$
$PM_c$	import price (domestic currency)
$PQ_c$	composite commodity price
$PVA_a$	value-added price (factor income per unit of activity)
$PX_c$	aggregate producer price for commodity
$PXAC_{ac}$	producer price of commodity $c$ for activity $a$
$QA_a$	quantity (level) of activity
$QD_c$	quantity sold domestically of domestic output

$QE_c$	quantity of exports
$QF_{fa}$	quantity demanded of factor $f$ from activity $a$
$QG_c$	government consumption demand for commodity $c$
$QH_{ch}$	quantity consumed of commodity $c$ by household $h$
$QHA_{ach}$	quantity of household home consumption of commodity $c$ from activity $a$ for household $h$
$QINTA_a$	quantity of aggregate intermediate input
$QINT_{ca}$	quantity of commodity $c$ as intermediate input to activity $a$
$QINV_c$	quantity of investment demand for commodity $c$
$QM_c$	quantity of imports of commodity $c$
$QQ_c$	quantity of goods supplied to domestic market (composite supply)
$QT_c$	quantity of commodity demanded as trade input
$QVA_a$	quantity of (aggregate) value-added
$QX_c$	aggregated marketed quantity of domestic output of commodity $c$
$QXAC_{ac}$	quantity of marketed output of commodity $c$ from activity $a$
$TABS$	total nominal absorption
$TINS_i$	direct tax rate for institution $i$ ( $i \in \text{INSDNG}$ )
$TRII_{ii'}$	transfers from institution $i'$ to $i$ (both in the set INSDNG)
$WF_f$	average price of factor $f$
$YF_f$	income of factor $f$
$YG$	government revenue
$YI_i$	income of domestic nongovernment institution $i$
$YIF_{if}$	income to domestic institution $i$ from factor $f$

## EQUATIONS

### Price Block

Import price

$$PM_c = pwm_c \cdot (1 + tm_c) \cdot EXR + \sum_{c' \in CT} PQ_{c'} \cdot icm_{c'c}$$

$$\begin{bmatrix} import \\ price \\ (LCU) \end{bmatrix} = \begin{bmatrix} import \\ price \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} tariff \\ \cdot \\ adjust- \\ ment \end{bmatrix} \cdot \begin{bmatrix} exchange rate \\ (LCU per \\ FCU) \end{bmatrix} + \begin{bmatrix} cost of trade \\ inputs per \\ import unit \end{bmatrix}$$

Export price

$$PE_c = pwe_c \cdot (1 - te_c) \cdot EXR - \sum_{c' \in CT} PQ_{c'} \cdot ice_{c'c}$$

$$\begin{bmatrix} export \\ price \\ (LCU) \end{bmatrix} = \begin{bmatrix} export \\ price \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} tariff \\ \cdot \\ adjust- \\ ment \end{bmatrix} \cdot \begin{bmatrix} exchange rate \\ (LCU per \\ FCU) \end{bmatrix} - \begin{bmatrix} cost of trade \\ inputs per \\ export unit \end{bmatrix}$$

Demand price of  
domestic nontraded goods

$$PDD_c = PDS_c + \sum_{c' \in CT} PQ_{c'} \cdot icd_{c'c}$$

$$\begin{bmatrix} domestic \\ demand \\ price \end{bmatrix} = \begin{bmatrix} domestic \\ supply \\ price \end{bmatrix} + \begin{bmatrix} cost of trade \\ inputs per \\ unit of \\ domestic sales \end{bmatrix}$$

Absorption

$$PQ_c \cdot (1 - tg_c) \cdot QD_c = PDD_c \cdot QD_c + PM_c \cdot QM_c$$

$c \in (CD \cup CM)$

$$\begin{bmatrix} absorption \\ (at demand \\ prices net of \\ sales tax) \end{bmatrix} = \begin{bmatrix} domestic demand price \\ times \\ domestic sales quantity \end{bmatrix} + \begin{bmatrix} import price \\ times \\ import quantity \end{bmatrix}$$

Marketed output value

$$PX_c \cdot QX_c = PDS_c \cdot QD_c + PE_c \cdot QE_c$$

$$\begin{bmatrix} producer price \\ times marketed \\ output quantity \end{bmatrix} = \begin{bmatrix} domestic supply price \\ times \\ domestic sales quantity \end{bmatrix} + \begin{bmatrix} export price \\ times \\ export quantity \end{bmatrix}$$

Activity price  
input price

$$PA_a = \sum_{c \in C} PXAC_{a,c} \cdot \theta_{a,c} \quad (6)$$

$$\begin{bmatrix} activity \\ price \end{bmatrix} = \begin{bmatrix} producer\ prices \\ times\ yields \end{bmatrix}$$

Aggregate intermediate  
input price

$$PINTA_a = \sum_{c \in C} PQ_c \cdot i ca_{c,a} \quad (7)$$

$$\begin{bmatrix} aggregate \\ intermediate \\ input\ price \end{bmatrix} = \begin{bmatrix} intermediate\ input\ cost \\ per\ unit\ of\ aggregate \\ intermediate\ input \end{bmatrix}$$

Activity revenue  
and costs

$$PA_u \cdot (1 - ta_u) \cdot QA_u = PVA_u \cdot QVA_u + PINTA_u \cdot QINTA_u \quad (8)$$

$$\begin{bmatrix} activity\ price \\ (net\ of\ taxes) \\ times\ activity\ level \end{bmatrix} = \begin{bmatrix} value-added \\ price\ times \\ quantity \end{bmatrix} + \begin{bmatrix} aggregate \\ intermediate \\ input\ price\ times \\ quantity \end{bmatrix}$$

Consumer price index

$$\overline{CPI} = \sum_{c \in C} PQ_c \cdot cwts_c \quad (9)$$

$$\begin{bmatrix} consumer \\ price\ index \end{bmatrix} = \begin{bmatrix} prices\ times \\ weights \end{bmatrix}$$

Producer price index for  
nontraded market output

$$DPI = \sum_{c \in C} PDS_c \cdot dwts_c \quad (10)$$

$$\begin{bmatrix} producer\ price\ index \\ for\ non-traded\ outputs \end{bmatrix} = \begin{bmatrix} prices\ times \\ weights \end{bmatrix}$$

## Production and Trade Block

CES technology: Activity production function

$$\underline{Q}A_a = \alpha_a^a \cdot \left( \delta_a^a \cdot \underline{Q}VA_a^{-\rho_a^a} + (1 - \delta_a^a) \cdot \underline{Q}INTA_a^{-\rho_a^a} \right)^{-\frac{1}{\rho_a^a}} \quad (11)$$

$$\begin{bmatrix} activity \\ level \end{bmatrix} = CES \begin{bmatrix} quantity of aggregate value added, \\ quantity of aggregate intermediate input \end{bmatrix}$$

CES technology: Value-added intermediate-input quantity ratio

$$\frac{\underline{Q}VA_a}{\underline{Q}INTA_a} = \left( \frac{PINTA_a}{PV A_a} \cdot \frac{\delta_a^a}{1 - \delta_a^a} \right)^{-\frac{1}{1+\rho_a^a}} \quad a \in ACES \quad (12)$$

$$\begin{bmatrix} value-added - \\ intermediate- \\ input quantity \\ ratio \end{bmatrix} = f \begin{bmatrix} intermediate-input : \\ value-added \\ price ratio \\ \end{bmatrix}$$

Leontief technology:  
Demand for aggregate value-added

$$\underline{Q}VA_a = iv a_a \cdot \underline{Q}A_a \quad a \in ALEO \quad (13)$$

$$\begin{bmatrix} demand for \\ value added \end{bmatrix} = f \begin{bmatrix} activity \\ level \end{bmatrix}$$

Leontief technology:  
Demand for aggregate intermediate input

$$\underline{Q}INTA_a = int a_a \cdot \underline{Q}A_a \quad a \in ALEO \quad (14)$$

$$\begin{bmatrix} demand for aggregate \\ intermediate input \end{bmatrix} = f \begin{bmatrix} activity \\ level \end{bmatrix}$$

Value-added and factor demands

$$\underline{Q}VA_a = \alpha_a^{va} \cdot \left( \sum_{f \in F} \delta_{f,a}^{va} \cdot \underline{Q}F_{f,a}^{-\rho_a^{va}} \right)^{-\frac{1}{\rho_a^{va}}} \quad a \in A \quad (15)$$

$$\begin{bmatrix} quantity of aggregate \\ value added \end{bmatrix} = CES \begin{bmatrix} factor \\ inputs \end{bmatrix}$$

$$\text{Factor demand} \quad WF_f \cdot \overline{WFDIST}_{f,a} = PVA_a \cdot (1 - \eta a_a) \cdot QVA_a \cdot \left( \sum_{f \in F^*} \delta_{f,a}^{va} \cdot QF_{f,a}^{-\rho_a^{w-1}} \right)^{-1} \cdot \delta_{f,a}^{va} \cdot QF_{f,a}^{-\rho_a^{w-1}} \quad a \in A, f \in F \quad (16)$$

$$\begin{bmatrix} \text{marginal cost of} \\ \text{factor } f \text{ in activity } a \end{bmatrix} = \begin{bmatrix} \text{marginal revenue product} \\ \text{of factor } f \text{ in activity } a \end{bmatrix}$$

$$\text{Disaggregated intermediate input demand} \quad QINT_{c,a} = i c a_{c,a} \cdot QINTA_a \quad (17)$$

$$\begin{bmatrix} \text{intermediate demand} \\ \text{for commodity } c \\ \text{from activity } a \end{bmatrix} = f \begin{bmatrix} \text{aggregate intermediate} \\ \text{input quantity} \\ \text{for activity } a \end{bmatrix}$$

$$\text{Commodity production and allocation} \quad QXAC_{a,c} + \sum_{h \in H} QH_{a,c,h} = \theta_{a,c} \cdot QA_a \quad (18)$$

$$\begin{bmatrix} \text{marketed quantity} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix} + \begin{bmatrix} \text{household home} \\ \text{consumption} \\ \text{of commodity } c \end{bmatrix} = \begin{bmatrix} \text{production} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix}$$

$$\text{Output aggregation function} \quad QX_c = \alpha_c^{ac} \cdot \left( \sum_{a \in A} \delta_{a,c}^{ac} \cdot QXAC_{a,c}^{-\rho_c^{ac}} \right)^{\frac{1}{\rho_c^{ac}-1}} \quad c \in CX \quad (19)$$

$$\begin{bmatrix} \text{aggregate} \\ \text{marketed} \\ \text{production of} \\ \text{commodity } c \end{bmatrix} = CES \begin{bmatrix} \text{activity-specific} \\ \text{marketed} \\ \text{production of} \\ \text{commodity } c \end{bmatrix}$$

$$\text{First-order condition for output aggregation function} \quad PX_{a,c} = PX_c \cdot QX_c \left( \sum_{a \in A'} \delta_{a,c}^{ac} \cdot QXAC_{a,c}^{-\rho_c^{ac}} \right)^{-1} \cdot \delta_{a,c}^{ac} \cdot QXAC_{a,c}^{-\rho_c^{ac}-1} \quad a \in A, c \in CX \quad (20)$$

$$\begin{bmatrix} \text{marginal cost of commodity } c \text{ from activity } a \end{bmatrix} = \begin{bmatrix} \text{marginal revenue product of} \\ \text{commodity } c \text{ from activity } a \end{bmatrix}$$

**Production and Trade Block (continued)**

Output transformation  
(CET) function

$$QX_c = \alpha'_c \cdot \left( \delta'_c \cdot QE_c^{\rho'_c} + (1 - \delta'_c) \cdot QD_c^{\rho'_c} \right)^{\frac{1}{\rho'_c}}$$

aggregate marketed  
domestic output

$$\left[ \begin{array}{c} \text{export quantity, domestic} \\ \text{sales of domestic output} \end{array} \right]$$

$c \in (CE \cap CD)$  (21)

$$\frac{QE_c}{QD_c} = \left( \frac{PE_c}{PDS_c} \cdot \frac{I - \delta'_c}{\delta'_c} \right)^{\frac{1}{\rho'_c}}$$

Export-domestic supply ratio

$$\left[ \begin{array}{c} \text{export-domestic} \\ \text{supply ratio} \end{array} \right] = f \left[ \begin{array}{c} \text{export-domestic} \\ \text{price ratio} \end{array} \right]$$

Output transformation for  
non-exported commodities

$$QX_c = QD_c + QE_c$$

$$\left[ \begin{array}{c} \text{aggregate} \\ \text{marketed} \\ \text{domestic output} \end{array} \right] = \left[ \begin{array}{c} \text{domestic market} \\ \text{output for} \\ c \in (CD \cap CEN) \end{array} \right] + \left[ \begin{array}{c} \text{exports [for} \\ c \in (CE \cap CDN)] \end{array} \right]$$

$c \in (CD \cap CEN)$   
 $\cup$   
 $(CE \cup CDN)$  (23)

$$QQ_c = \alpha^q_c \cdot \left( \delta^q_c \cdot QM_c^{\rho^q_c} + (1 - \delta^q_c) \cdot QD_c^{\rho^q_c} \right)^{\frac{1}{\rho^q_c}}$$

Composite supply  
(Armington) function

$$\left[ \begin{array}{c} \text{composite} \\ \text{supply} \end{array} \right] = f \left[ \begin{array}{c} \text{import quantity, domestic} \\ \text{use of domestic output} \end{array} \right]$$

$c \in (CM \cap CD)$  (24)

$$\frac{QM_c}{QD_c} = \left( \frac{PDD_c}{PM_c} \cdot \frac{\delta^q_c}{1 - \delta^q_c} \right)^{\frac{1}{1 + \rho^q_c}}$$

Import-domestic demand ratio

$$\left[ \begin{array}{c} \text{import-domestic} \\ \text{demand ratio} \end{array} \right] = f \left[ \begin{array}{c} \text{domestic-import} \\ \text{price ratio} \end{array} \right]$$

$c \in (CM \cap CD)$  (25)

Composite supply for  
non-imported outputs  
and nonproduced imports

$$QQ_c = QD_c + QM_c$$

$$\begin{aligned} \left[ \begin{array}{c} \text{composite} \\ \text{supply} \end{array} \right] &= \left[ \begin{array}{c} \text{domestic use of} \\ \text{marketed domestic} \\ \text{output [for} \\ \text{CD]} \end{array} \right] + \left[ \begin{array}{c} \text{imports [for} \\ \text{CM] and CMN]} \\ \text{CDN]} \end{array} \right] \\ &\quad \cup \\ &\quad \left[ \begin{array}{c} \text{output [for} \\ \text{CM \cap CMN]} \end{array} \right] \end{aligned}$$

Demand for  
transactions services

$$QT_c = \sum_{c' \in C'} (icm_{cc'} \cdot QM_{c'} + ice_{cc'} \cdot QE_{c'} + icd_{cc'} \cdot QD_{c'})$$

$$\left[ \begin{array}{c} \text{demand for} \\ \text{transactions} \\ \text{services} \end{array} \right] = \left[ \begin{array}{c} \text{sum of demands} \\ \text{for imports, exports,} \\ \text{and domestic sales} \end{array} \right]$$

### Institution Block

Factor income

$$YF_f = \sum_{a \in A} WF_f \cdot \overline{WF DIST}_{fa} \cdot QF_{fa}$$

$$\left[ \begin{array}{c} \text{income of} \\ \text{factor } f \\ \text{from factor } f \end{array} \right] = \left[ \begin{array}{c} \text{sum of activity payments} \\ \text{(activity-specific wages} \\ \text{times employment levels)} \end{array} \right]$$

$$\begin{aligned} YIF_{if} &= shif_{if} \cdot \left[ (1 - tf_f) \cdot YF_f - transfr_{rowf} \cdot EXR \right] \\ \left[ \begin{array}{c} \text{income of} \\ \text{institution } i \\ \text{from factor } f \end{array} \right] &= \left[ \begin{array}{c} \text{share of income} \\ \text{of factor } f \text{ to} \\ \text{institution } i \end{array} \right] \cdot \left[ \begin{array}{c} \text{income of factor } f \\ \text{(net of tax and} \\ \text{transfer to RoW)} \end{array} \right] \end{aligned}$$

Institutional factor incomes

$$\left[ \begin{array}{c} \text{income of} \\ \text{institution } i \end{array} \right] = \left[ \begin{array}{c} \text{factor} \\ \text{income} \end{array} \right] + \left[ \begin{array}{c} \text{transfers} \\ \text{from other domestic} \\ \text{institutions} \end{array} \right] + \left[ \begin{array}{c} \text{transfers} \\ \text{from} \\ \text{non-government} \end{array} \right] + \left[ \begin{array}{c} \text{transfers} \\ \text{from} \\ \text{government} \end{array} \right]$$

$$\begin{aligned} YI_i &= \sum_{f \in F} YIF_{if} + \sum_{i' \in INSDNG} TRII_{ii'} + transfr_{igov} \cdot \overline{CPI} + transfr_{irow} \cdot EXR \\ \left[ \begin{array}{c} \text{income of} \\ \text{institution } i \end{array} \right] &= \left[ \begin{array}{c} \text{factor} \\ \text{income} \end{array} \right] + \left[ \begin{array}{c} \text{transfers} \\ \text{from other domestic} \\ \text{institutions} \end{array} \right] + \left[ \begin{array}{c} \text{transfers} \\ \text{from} \\ \text{non-government} \end{array} \right] + \left[ \begin{array}{c} \text{transfers} \\ \text{from} \\ \text{government} \end{array} \right] \end{aligned}$$

Income of domestic,  
nongovernment institutions

$$i \in INSDNG$$

## Institution Block (continued)

Intra-institutional transfers

$$TRII_{i,i'} = shii_{i,i'} \cdot (1 - MPS_{i'}) \cdot (1 - TIN_{i'}) \cdot Y_{i'} \quad i \in INSDNG \\ transfer from \begin{bmatrix} share of net income \\ of institution i' \end{bmatrix} = \begin{bmatrix} income of institution \\ i', net of savings and \\ direct taxes \end{bmatrix} \quad i' \in INSDNG' \quad (31)$$

Household consumption expenditure

$$EH_h = \left( 1 - \sum_{i \in INSDNG} shii_{i,h} \right) \cdot (1 - MPS_h) \cdot (1 - TIN_h) \cdot Y_{i'} \quad h \in H \quad (32)$$

$$\begin{bmatrix} household income \\ disposable for \\ consumption \end{bmatrix} = \begin{bmatrix} household income, net of direct \\ taxes, savings, and transfers to \\ other non-government institutions \end{bmatrix}$$

Household consumption demand for marketed commodities

$$PQ_c \cdot QH_{c,h} = PQ_c \cdot \gamma_{c,h}^m + \beta_{c,h}^h \left( EH_h - \sum_{c \in C} PQ_c \cdot \gamma_{c,h}^m - \sum_{a \in A} \sum_{c \in C} PXAC_{a,c} \cdot \gamma_{a,c,h}^h \right) \quad c \in C \quad (33)$$

$$\begin{bmatrix} household consumption \\ spending on market \\ commodity c \end{bmatrix} = f \begin{bmatrix} total household consumption \\ spending, market price of c, and other \\ commodity prices (market and home) \end{bmatrix} \quad h \in H$$

Household consumption demand for home commodities

$$PXAC_{a,c} \cdot QHA_{a,c,h} = PXAC_{a,c} \cdot \gamma_{a,c,h}^h + \beta_{a,c,h}^h \left( EH_h - \sum_{c' \in C} PQ_{c'} \cdot \gamma_{c',h}^m - \sum_{a \in A} \sum_{c \in C} PXAC_{a,c} \cdot \gamma_{a,c,h}^h \right) \quad a \in A \quad (34)$$

$$\begin{bmatrix} household consumption \\ spending on home commodity \\ c from activity a \end{bmatrix} = f \begin{bmatrix} total household consumption spending, \\ producer price, and other \\ commodity prices (market and home) \end{bmatrix} \quad h \in H$$

Investment demand

$$QINV_c = \overline{IADJ} \cdot \overline{qinv}_c \quad c \in CINV \quad (35)$$

$$\begin{bmatrix} fixed investment \\ demand for \\ commodity c \end{bmatrix} = \begin{bmatrix} adjustment factor \\ times \\ base-year fixed \\ investment \end{bmatrix}$$

$$QG_c = \overline{GADJ} \cdot \overline{qg_c}$$

$$\begin{bmatrix} government \\ consumption \\ demand \end{bmatrix}_c = \begin{bmatrix} adjustment factor \\ times \\ base-year government \\ consumption \end{bmatrix}_c$$
(36)

Government  
consumption demand

$$\text{Government revenue}$$

$$YG = \sum_{i \in \text{INSDNG}} TINS_i \cdot YI_i + \sum_{f \in F} tf_f \cdot YF_f + \sum_{a \in A} ta_a \cdot PVA_a \cdot QVA_a$$

$$+ \sum_{a \in A} ta_a \cdot PA_a \cdot QA_a + \sum_{c \in CM} tm_c \cdot PWM_c \cdot QM_c \cdot EXR + \sum_{e \in CE} te_e \cdot PWE_e \cdot QE_e \cdot EXR$$

$$+ \sum_{c \in C} tq_c \cdot PQ_c \cdot QQ_c + \sum_{f \in F} YIF_{gov,f} + transfr_{gov, row} \cdot EXR$$
(37)

$$\begin{bmatrix} government \\ revenue \end{bmatrix} = \begin{bmatrix} direct taxes \\ from institutions \end{bmatrix} + \begin{bmatrix} value-added \\ factors \end{bmatrix} + \begin{bmatrix} tax \end{bmatrix}$$

$$+ \begin{bmatrix} activity \\ tax \end{bmatrix} + \begin{bmatrix} import \\ tariffs \end{bmatrix} + \begin{bmatrix} export \end{bmatrix} + \begin{bmatrix} sales \\ taxes \end{bmatrix} + \begin{bmatrix} factor \\ tax \end{bmatrix} + \begin{bmatrix} transfers \\ from income \end{bmatrix} + \begin{bmatrix} transfers \\ RoW \end{bmatrix}$$

$$\text{Government expenditures}$$

$$EG = \sum_{c \in C} PQ_c \cdot QG_c + \sum_{i \in \text{INSDNG}} transfr_{i, gov} \cdot \overline{CPI}$$

$$\begin{bmatrix} government \\ spending \end{bmatrix} = \begin{bmatrix} government \\ consumption \end{bmatrix} + \begin{bmatrix} transfers to domestic \\ non-government institutions \end{bmatrix}$$
(38)

**System**  
**Constraint Block**  
Factor market

$$\sum_{a \in A} QF_{f,a} = \overline{QFS}_f$$

$$\begin{bmatrix} demand \\ for \end{bmatrix}_{factor f} = \begin{bmatrix} supply of \\ factor f \end{bmatrix}$$
(39)

$f \in F$

## System Constraint Block (continued)

Current account balance  
for rest of the world  
(in foreign currency)

$$\text{Composite commodity markets} \quad \begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = \begin{bmatrix} \text{intermediate} \\ \text{use} \end{bmatrix} + \begin{bmatrix} \text{household} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix} + \begin{bmatrix} \text{trade} \\ \text{input use} \end{bmatrix} \quad (40)$$

Government balance  
for rest of the world  
(in foreign currency)

$$\text{Current account balance} \quad \sum_{c \in CM} pwm_c \cdot QM_c + \sum_{f \in F} transfr_{row,f} = \sum_{c \in CE} pwe_c \cdot QE_c + \sum_{i \in INSD} transfr_{i, \text{row}} + \overline{FSAV} \quad (41)$$

$$\begin{bmatrix} \text{import} \\ \text{spending} \end{bmatrix} + \begin{bmatrix} \text{factor} \\ \text{transfers} \end{bmatrix} + \begin{bmatrix} \text{to RoW} \end{bmatrix} = \begin{bmatrix} \text{export} \\ \text{revenue} \end{bmatrix} + \begin{bmatrix} \text{institutional} \\ \text{transfers} \end{bmatrix} + \begin{bmatrix} \text{foreign} \\ \text{savings} \end{bmatrix}$$

Direct institutional tax rates

$$YG = EG + GSAV \quad (42)$$

$$\begin{bmatrix} \text{government} \\ \text{revenue} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{expenditures} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{savings} \end{bmatrix}$$

$$TINS_i = \overline{tins}_i \cdot \left( 1 + \overline{TNSADJ} \cdot tins01_i \right) + \overline{DTINS} \cdot tins01_i \quad i \in INSDNG \quad (43)$$

$$\begin{bmatrix} \text{direct tax} \\ \text{rate for} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{base rate adjusted} \\ \text{for scaling for} \\ \text{selected institutions} \end{bmatrix} + \begin{bmatrix} \text{point change} \\ \text{for selected} \\ \text{institutions} \end{bmatrix}$$

$$MPS_i = \overline{mps}_i \cdot \left( 1 + \overline{MPSADU} \cdot mps01_i \right) + DMPS \cdot mps01_i \quad i \in INSDNG \quad (44)$$

$$\begin{bmatrix} \text{savings} \\ \text{rate for} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{base rate adjusted} \\ \text{for scaling for} \\ \text{selected institutions} \end{bmatrix} + \begin{bmatrix} \text{point change} \\ \text{for selected} \\ \text{institutions} \end{bmatrix}$$

Savings-Investment Balance

$$\sum_{i \in INSDNG} MPS_i \cdot (1 - TINS) \cdot YI_i + GSAV + EXR \cdot FSAV = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c$$

$$\begin{bmatrix} non-government \\ government \\ savings \end{bmatrix} + \begin{bmatrix} foreign \\ savings \end{bmatrix} = \begin{bmatrix} fixed \\ investment \\ stock \\ change \end{bmatrix}$$
(45)

Total absorption

$$TABS = \sum_{h \in H} \sum_{c \in C} PQ_c \cdot QH_{ch} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC_{ach} \cdot QH_{ach} + \sum_{c \in C} PQ_c \cdot QG_c$$

$$+ \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c$$

$$\begin{bmatrix} total \\ absorption \end{bmatrix} = \begin{bmatrix} household \\ market \\ consumption \end{bmatrix} + \begin{bmatrix} household \\ home \\ consumption \end{bmatrix} + \begin{bmatrix} government \\ consumption \end{bmatrix} + \begin{bmatrix} fixed \\ investment \\ stock \\ change \end{bmatrix}$$
(46)

Ratio of investment  
to absorption

$$INVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c$$

$$\begin{bmatrix} investment- \\ absorption \\ ratio \end{bmatrix} \cdot \begin{bmatrix} total \\ absorption \end{bmatrix} = \begin{bmatrix} fixed \\ investment \\ stock \\ change \end{bmatrix}$$
(47)

Ratio of government  
consumption to absorption

$$GOVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QG_c$$

$$\begin{bmatrix} government \\ consumption \\ absorption \\ ratio \end{bmatrix} \cdot \begin{bmatrix} total \\ absorption \end{bmatrix} = \begin{bmatrix} government \\ consumption \end{bmatrix}$$
(48)