EQUATION SYSTEM FOR UNCONSTRAINED SAM MULTIPLIER

We replace actual numbers in the SAM with the following symbols.

	Activities		Commodities		Factors	House-holds	Exogenous demand	Total
	A1	A2	C1	C2	F	H	E	
A1			X_1					X_1
A2				X_2				X_2
C1	Z_{11}	Z_{12}				C_1	E_1	Z_1
C2	Z_{21}	Z_{22}				C_2	E_2	Z_2
F	V_1	V_2						V
H					$V_1 + V_2$			Y
E			L_1	L_2		S		E
Total	X_1	X_2	Z_1	Z_2	V	Y	E	

We divide columns by their total to derive the coefficients matrix (M-matrix). Note that the M-matrix excludes the exogenous components of demand.

	Activities		Commodities		Factors	House-holds	Exogenous demand	Total
	A1	A2	C1	C2	F	H	E	
A1			$b_1 = X_1/Z_1$					X_1
A2				$b_2 = X_2/Z_2$				X_2
C1	$a_{11}=Z_{11}/X_1$	$a_{12}=Z_{12}/X_2$				$c_1 = C_1/Y$	E_1	Z_1
C2	$a_{21}=Z_{21}/X_1$	$a_{22}=Z_{22}/X_2$				$c_2 = C_2/Y$	E_2	Z_2
F	$v_1=V_1/X_1$	$v_2=V_2/X_2$						V
H					1			Y
E			$l_1 = L_1/Z_1$	$l_2 = L_2/Z_2$		$s = S/Y$		E
Total	1	1	1	1	1	1	E	

<u>Values</u>

- X Gross output of each activity (i.e., X_1 and X_2)
- Z Total demand for each commodity (i.e., Z_1 and Z_2)
- V Total factor income (equal to household income)
- Y Total household income (equal to total factor income)
- E Exogenous components of demand (i.e., government, investment, and exports)

<u>Shares</u>

- a Technical coefficients (i.e., input or intermediate shares in production)
- b Share of domestic output in total demand
- v Share of value-added or factor income in gross output
- l Share of the value of total demand from imports or commodity taxes
- c Household consumption expenditure shares
- s Household savings rate (i.e., savings as a share of household income)

So we can now derive equations representing the relationships in the SAM. We start with the simple demand equations.

$$Z_1 = a_{11}X_1 + a_{12}X_2 + c_1Y + E_1$$
$$Z_2 = a_{21}X_1 + a_{22}X_2 + c_2Y + E_2 \tag{A1}$$

Total demand = intermediate demand + household demand + exogenous demand

From the SAM, we know that domestic production X is only part of total demand Z.

$$X_1 = b_1Z_1 \quad \text{and} \quad X_2 = b_2Z_2$$

We know that household income Y depends on the share each factor earns in each sector.

$$Y = v_1X_1 + v_2X_2 \quad \text{or} \quad Y = v_1b_1Z_1 + v_2b_2Z_2$$

Now we replace Xs and Vs in Equation A1.

$$Z_1 = a_{11}b_1Z_1 + a_{12}b_2Z_2 + c_1(v_1b_1Z_1 + v_2b_2Z_2) + E_1$$
$$Z_2 = a_{21}b_1Z_1 + a_{22}b_2Z_2 + c_2(v_1b_1Z_1 + v_2b_2Z_2) + E_2$$

We move everything except for E onto the left-hand side.

$$Z_1 - a_{11}b_1Z_1 - c_1v_1b_1Z_1 - a_{12}b_2Z_2 - c_1v_2b_2Z_2 = E_1$$
$$-a_{21}b_1Z_1 - c_2v_1b_1Z_1 + Z_2 - a_{22}b_2Z_2 - c_2v_2b_2Z_2 = E_2$$

We group Zs together.

$$(1 - a_{11}b_1 - c_1v_1b_1)Z_1 + (-a_{12}b_2 - c_1v_2b_2)Z_2 = E_1$$
$$(-a_{21}b_1 - c_2v_1b_1)Z_1 + (1 - a_{22}b_2 - c_2v_2b_2)Z_2 = E_2 \tag{A2}$$

We express Equation A2 in matrix format.

$$\begin{pmatrix} 1 - a_{11}b_1 - c_1v_1b_1 & -a_{12}b_2 - c_1v_2b_2 \\ -a_{21}b_1 - c_2v_1b_1 & 1 - a_{22}b_2 - c_2v_2b_2 \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \tag{A3}$$

The first term in Equation A3 is the identity matrix (I) minus the coefficient matrix (M).

$$\begin{pmatrix} 1 - a_{11}b_1 - c_1v_1b_1 & -a_{12}b_2 - c_1v_2b_2 \\ -a_{21}b_1 - c_2v_1b_1 & 1 - a_{22}b_2 - c_2v_2b_2 \end{pmatrix} = I - M$$

If we rename the other two vectors Z and E then we can simply Equation A3.

$$(I - M)Z = E \tag{A4}$$

Rearranging, we get the final multiplier equation.

$$Z = (I - M)^{-1}E \tag{A5}$$

Total demand = multiplier matrix × exogenous demand

This tells us that when exogenous demand [E] increases, then after you have taken all the direct and indirect multiplier effects into account [$(I\text{-}M)^{-1}$], you will end up with a final increase in total demand equal to Z.